



Faculty of Health, Science and Technology
Physics

Syllabus

Symmetry - mathematical structures and applications

Course Code: FYAD11

Course Title: Symmetry - mathematical structures and applications
Symmetri - matematiska strukturer och tillämpningar

Credits: 7.5

Degree Level: Master's level

Progressive Specialisation: Second cycle, has only first-cycle course/s as entry requirements (A1N)

Major Field of Study:

FYA (Physics)

MAA (Mathematics)

TKA (Engineering Physics)

Course Approval

The syllabus was approved by the Faculty of Health, Science and Technology 2024-12-03, and is valid from the Autumn semester 2025 at Karlstad University.

Prerequisites

Mathematics and Physics 105 ECTS credits, plus upper secondary level English 6, or equivalent

Learning Outcomes

The aim of the course is for students to acquire substantial knowledge of mathematical structures and methods used to describe symmetries in classical physics and quantum physics, and their applications in nuclear physics, molecular physics, and condensed matter physics.

Upon completion of the course, students should be able to:

- identify discrete and continuous symmetries in physical systems and describe them using relevant mathematical structures,
- provide definitions for the central mathematical structures that are necessary for the study of symmetries and give an account of central aspects of their structural theory and representation theory,
- summarise the main steps of classifying finite-dimensional complex simple Lie algebras,
- apply symmetry methods to issues in nuclear physics, molecular physics, and condensed matter physics, especially spectroscopy, crystallography, and the theory of molecular vibration,
- handle the graphical tensor calculus to visualise algebraic structures and their representation theory and perform proofs, and
- formulate processes in quantum mechanics using the graphical tensor calculus.

Content

Instruction is in the form of lectures, calculation exercises, and individual projects.

The lectures cover the following themes:

- definition, examples, and aspects of structure theory for various algebraic structures: finite groups, associative algebra, Hopf algebra, Frobenius algebra, finite-dimensional Lie groups and Lie algebra, quantum groups, and supersymmetries
- the basics of representation theory for groups and Lie algebras, including construction of character table for finite groups, factorisation of the tensor product of representations, basic functions of irreducible representations, PBW theorem and Weyl's character formula,
- classification of finite-dimensional complex simple Lie algebras and introduction to Kac-Moody and affine Lie algebras,
- quantum mechanics applications, including Bloch's theorem, symmetry-adapted wave functions of molecular orbitals and crystalline fields splitting of nuclear orbitals,
- spectroscopy applications, including Unsold's theorem and electrical/magnetic dipole transfers,
- applications in molecular vibrations and crystalline symmetries,
- tensor categories, graphical tensor calculus, application of the graphical tensor calculus to visualise algebraic relations, and graphical proof,
- applications of the graphical calculus in quantum mechanics and quantum information.

Reading List

See separate document.

Examination

Assessment is based on hand-in assignments, written and oral presentations of projects, and an individual oral exam.

If students have a decision from Karlstad University entitling them to Targeted Study Support due to a documented disability, the examiner has the right to give such students an adapted examination or to examine them in a different manner.

Grades

One of the grades Distinction (VG), Pass (G), or Fail (U) is awarded in the examination of the course. For students in Engineering, one of the grades 5 (Pass with Distinction), 4 (Pass with

Some Distinction), 3 (Pass), or U (Fail) is awarded in the examination of the course.

Quality Assurance

Follow-up relating to learning conditions and goal-fulfilment takes place both during and upon completion of the course in order to ensure continuous improvement. Course evaluation is partly based on student views and experiences obtained in accordance with current regulations and partly on other data and documentation. Students will be informed of the result of the evaluation and of any measures to be taken.

Course Certificate

A course certificate will be provided upon request.

Additional information

The local regulations for studies at the Bachelor and Master levels at Karlstad University stipulate the obligations and rights of students and staff.

The course FYAD11 cannot be included in the same degree programme as the course FYAD08.