Problem 1. Demand

Bengt’s utility function is \( U(x_1, x_2) = x_1 + \ln x_2 \)

\( x_1 \) - stamps
\( x_2 \) - beer

Bengt’s budget \( p_1 x_1 + p_2 x_2 = m \)

\( p_1 \) – price of stamps
\( p_2 \) – price of beer
\( m \) – Bengt’s budget

a) What is Bengt’s demand for beer and stamps?
b) Is it true that Bengt would spend every krona in additional income on stamps?
c) What happens to demand when Bengt’s income changes (i.e. find the income elasticity)?
d) What happens to demand when \( p_1 \) and \( p_2 \) increase (i.e. find the price elasticities)?

Problem 2. Demand

Jan has fallen on hard times. His income per week is 400 kr, spending 200 kr on food and 200 kr on all other goods. However, he is also receiving a social allowance in the form of 10 food stamps per week. The coupons can be exchanged for 10 kr worth of food, and he only has to pay 5 kr for such coupons. Show the budget line with and without the food stamps. If Jan has homothetic preferences, how much more food will he buy when he receives the food stamps?

Problem 3. Demand

Find the demand functions for the individuals below, the budget constraint is \( p_1 x_1 + p_2 y = m \)

Bill: \( U(x_1, x_2) = x_2 \left( \frac{3}{x_2} \right) \)

Buster: \( U(x, y) = x^{2/5} y^{3/5} \)

Ben: \( U(x, y) = (x+1)^2 (y+2)^3 \)

Barbara: \( U(x_1, x_2) = 3x + 2y \)

Beth: \( U(x, y) = \min\{x, y\} \)
Problem 4. Demand

Birgitta spends 150 SEK per month on coffee and buns at the cafeteria. A cup of coffee costs 15 SEK and a bun costs 10 SEK.

a) Write the equation for Birgitta’s cafeteria budget constraint and draw it in a diagram.

b) Assume that Birgitta never drinks coffee without eating one bun, and never eats buns without drinking coffee. How much of each will she consume? Draw some of her indifference curves.

c) What do we call goods that are always consumed in the same proportion?

Problem 1 Slutsky equation

Tomas is trying to figure out how to supplement the study allowances of 500 kr a week. He is considering a part-time job at a gas station. The wage is 50 kr per hour. His utility function is \( U(C, L) = C^*L \) where \( C \) is his consumption measured in SEK and \( L \) his leisure measured in hours. The amount of leisure time that he has left after allowing for necessary activities is 50 hours a week.

a. What is the monetary value of Tomas' endowment?


c. Set up the maximisation problem and decide optimal consumption and leisure.

d. Let \( Y \) = study allowance and \( T \) = total amount of leisure time. Express his demand for consumption as a function of study allowance and wage.

e. Express his supply function for labour as a function of study allowance and wage.

f. How many hours would Tomas work if he did not receive any study allowance?

Problem 2 Slutsky equation

Assume that the function \( U(x, y) = x^{0.3} y^{0.5} \) is the utility function of a person who consumes two goods in quantities \( x \) and \( y \), respectively.

The price of \( x \) is \( p_x = 5 \) and the price of \( y \) is \( p_y = 8 \)

This person’s income is \( m = 160 \).

a) Find the optimal consumption choice of this person.

b) Verify that at the optimum that you found the marginal rate of substitution equals the price ratio. Explain in terms of economic theory why this should be the case!

c) Assume that the price of \( x \) falls to \( p_x = 4 \).
i. Draw the old and the new budget constraints in a diagram. (Indicate at what values they intersect the axes).

ii. Calculate the person’s demand for x and y at the new price.

iii. Calculate the compensated income, m’.

iv. Decompose the change in demand for good x into a substitution and an income effect.

**Problem 1. Consumer’s surplus**
Mattias has quasilinear preferences and his demand function for books is \( B = 15 - 0.5p \).

a) Write the inverse demand function.

b) Mattias is currently consuming 10 books at a price of 10 kr. How much money would he be willing to pay to have this amount, rather than no books at all? What is his level of consumer’s surplus?

**Problem 2. Consumer’s surplus**
Suppose Birgitta has the utility function \( U = x_1^{0.1} x_2^{0.9} \). She has an income of 100 and \( P_1 = 1 \) and \( P_2 = 1 \). Calculate compensating and equivalent variation when the price of x1 increases to 2. Also, try to estimate the change in consumer’s surplus measured by the area below the demand function.

**Problem 3. Consumer’s surplus**
Explain the concept of “consumer surplus” in words and illustrate by a diagram.

**Problem 4. Consumer’s surplus**
The inverse demand curve (the demand curve but with p instead of q on the left hand side) is given by \( p(q) = 100 - 10q \). The consumer consumes five units of the good (q).

a) How much money would you have to pay to compensate her for reducing her consumption to zero? (The consumer is not paying anything for the goods.)

b) Suppose now that the consumer is buying the goods at a price of 50 per unit. If you now require her to reduce her purchases to zero, how much does she need to get compensated? Hint: The number you will find is the net consumer’s surplus.

**Problem 5. Consumer’s surplus**
New housing is planned in Karlstad but the location where it is to be built is used as a popular recreation area for people in neighbouring parts of the city. In order to decide whether to build or not, the city authorities want to make a survey to measure the decrease in welfare due the loss of
this recreation area. They are told by an economist that two measures are possible, compensating variation (CV) and equivalent variation (EV).

a) How should they formulate the question if they want to measure the compensating variation?

b) How should they phrase it if they want to measure the equivalent variation?

Problem 1. Market demand

Linus has a demand function \( q = 10 - 2p \)

a. What is the price elasticity of demand when the price is 3?

b. At what price is the elasticity of demand equal to -1?

c. Suppose that his demand function takes the general form \( q = a - bp \). Write down an algebraic expression for his elasticity of demand at an arbitrary price \( p \).

Problem 2. Market demand

The demand function is \( q(p) = (p+1)^2 \)

a. What is the price elasticity of demand?

b. At what price is the price elasticity of demand equal to minus one?

c. Write an expression for total revenue as a function of the price.

d. Answer a-c when the demand function takes the more general form \( q(p) = (p+a)b \) where \( a > 0 \) and \( b < -1 \).

Problem 3. Market demand

Find the price elasticity of demand for the following demand functions.

a) \( D(p)=30-6p \)
b) \( D(p)=60-p \)
c) \( D(p)=a-bp \)
d) \( D(p)=40p^{-2} \)
e) \( D(p)=Ap^{-b} \)
f) \( D(p)=(p+3)^2 \)

Problem 1. Equilibrium

Suppose we have the following demand and supply equations

\( D(p) = 200 - p \)
\( S(p) = 150 + p \)
a. What is the equilibrium price and quantity?

b. The government decides to restrict the industry to selling only 160 units by imposing a maximum price and rationing the good. What maximum price should the government impose?

c. The government doesn't want the firms in the industry to receive more than the minimum price that it would take to have them supply 160 units of the good. Therefore, they issue 160 ration coupons. If the ration coupons were freely bought and sold on the open market, what would be the equilibrium price of these coupons?

d. Calculate the dead-weight loss from restricting the supply of the goods. Will the dead-weight loss increase or decrease if the government would not allow the coupons to be sold on the open market?

Problem 2. Equilibrium

The demand curve is \( q_D = 100 - 5p \) and the supply curve is \( q_S = 5p \).

a. A quantity tax of 2 kr per unit is placed on the good. Calculate the dead-weight loss of the tax.

b. A value (ad valorem) tax of 20 % is placed on the good. Calculate the dead-weight loss of the tax.

Problem 3. Equilibrium

Assume that both demand and supply for a good are linear functions of its price:

\[
D(p) = a + bp, \quad a > 0, \quad b < 0 \\
S(p) = c + dp, \quad c < 0, \quad d > 0
\]

a) Draw curves that fit this description in a diagram.

b) Assume that a tax \( t \) per unit has to be paid by the consumer. Show the effects on demand, supply, equilibrium price, quantity consumed and consumer and producer welfare in your diagram.

c) Assume instead that an equally large tax has to be paid by the producer. What are the effects now on demand, supply, equilibrium price, quantity consumed and consumer and producer welfare. (Use a diagram to illustrate.)

Problem 1. Intertemporal choice

Suppose that a consumer has an endowment of 200.000 kr each period (period 1 and 2). He can borrow money at an interest rate of 200%, and he can lend money at a rate of 0%.

a. Illustrate his budget set.

b. The consumer is offered an investment that will change his endowment to \( m_1 = 300.000 \) and \( m_2 = 150.000 \). Would the consumer be better or worse off, or can't you tell?
c. If he is offered \( m_1 = 150,000 \) and \( m_2 = 300,000 \), is he better or worse off?

**Problem 2. Intertemporal choice**

Mainy Landin has an income of 200,000 kr this year and she expects an income of 110,000 kr next year. She can borrow and lend money at an interest rate of 10%. Consumption goods cost 1 kr and there is no inflation.

a. What is the present value of Mainy’s endowment?

b. What is the future value of Mainy’s endowment?

c. Suppose that Mainy has the utility function \( U = c_1c_2 \). Write down Mainy’s marginal rate of substitution.

d. Set this slope equal to the slope of the budget line and solve for the consumption in period 1 and 2. Will she borrow or save in the first period.

e. = d, but the interest rate is 20%. Will Mainy be better or worse off?

**Problem 1. Uncertainty**

Jonas Thern maximises expected utility:

\[ U(\pi_1, \pi_2, c_1, c_2) = \pi_1 c_1 + \pi_2 c_2 \]

Jonas’s friend Stefan Schwarz has offered to bet him 10,000 kr on the outcome of the toss of a coin. If the coin comes up head, Jonas must pay Stefan 10,000 kr, and if the coin comes up tails, Stefan must pay Jonas 10,000 kr. If Jonas doesn’t accept the bet, he will have 100,000 kr with certainty. Let Event 1 be "coin comes up heads”.

a. What is Jonas’s utility if he accepts the bet and if he decides not to bet? Does Jonas take the bet?

b. Answer the question in a, if the bet is 100,000 kr.

c. Answer the question if Jonas must pay Stefan 100,000 kr if he coin comes up head, but if the coin comes up tails Stefan must pay Jonas 500,000 kr.

d. Klas Ingesson would also like to gamble with Jonas. He is very intelligent and realises the nature of Jonas’ preferences. He offers him a bet that Jonas will take. Klas says: “If you loose you will give me 10,000 kr. If you win, I will give you .......?”

**Problem 2. Uncertainty**

Gabriel likes to gamble and his preferences are represented by the expected utility function

\[ U = \pi_1 c_1^2 + \pi_2 c_2^2 \]

Gabriel has not worked out very well, he only have 1,000 kr. Thomas shuffled a deck of cards and offered to bet Gabriel 200 kr that he would not cut a spade from the deck.
a. Show that Gabriel refuses the bet.
b. Would Gabriel accept the bet if they would bet 1,000 kr instead of 200 kr?
c. Sketch one of Gabriel’s indifference curves (let Event 2 be the event that a card drawn from a fair deck of cards is a spade)
d. On the same graph, sketch the indifference curve when the gamble is that he would not cut a black card from the deck.

Problem 3. Uncertainty
Consider an individual with an income of 100. She has the option of participating in a lottery where she can win 30 with a probability of 0.5, and loose 30 with a probability of 0.5. Would she participate if she is risk averse? What if she is a risk lover? Explain

Answers to the problems

Problem 1. Demand
a) Given prices, $p_1$ and $p_2$, find the quantities $x_1$ and $x_2$ which maximise Bengt’s utility!

Necessary condition:

$$MRS = \frac{MU_1}{MU_2} = \frac{\frac{p_1}{x_1}}{\frac{p_2}{x_2}}$$

$$MU_1 = 1 \quad \quad MU_2 = \frac{1}{x_2}$$

$$MRS = \frac{x_2}{p_2}$$

Therefore the optimum occurs when

$$x_2 = \frac{P_1}{P_2}$$

Money left to buy $x_1$ for:

$$m - p_2 x_2 = m - p_2 \frac{p_1}{p_2} = m - p_1$$

$$x_1 = \frac{m - p_1}{p_1} = \frac{m}{p_1} - 1$$

if $m > p_1$

b) Yes, if $m > p_1$ he won’t buy any more beer when $m$ increases.

c)-d)
Problem 2. Demand

\( y = 400 \)

10 food stamps per week, price 5 kr. Can be exchanged for 10 kr worth of food.

Old budget line: max 400 other and 400 food.

New budget line: max 400 other and 450 food, kink at 100 food and 350 other.

Homothetic preferences: The income expansion path is a straight line through origin.

Since Jan spent half his income on food, he will continue doing so. His income increases by 50 kr, thus he spends 25 kr more on food.

Problem 3. Demand

Bill:

\[
\frac{\partial u}{\partial x} = 2xy^3 \quad \frac{\partial u}{\partial y} = 2x^2y^2
\]

\[
\frac{2xy^3}{3x^2y^2} = \frac{p_x}{p_y} \quad \text{ (simplify)}
\]

\[
\frac{2y}{3x} = \frac{p_x}{p_y} \quad \text{ (solve for } y \text{)}
\]

\[
y = \frac{3p_x}{2p_y} \quad \text{ (insert this into the budget restriction)}
\]
\[ m = P_y \frac{3P_x x}{2P_y} + P_x x = \frac{3P_x x}{2} + P_x x = \frac{5}{2} P_x x \] (solve for x)

\[ x = \frac{2m}{5P_x} \] (insert this into the expression for y)

\[ y = \frac{3P_x 2m}{2P_y 5P_x} = \frac{6m}{10P_y} = \frac{3m}{5P_y} \]

We can also solve this by transforming the utility function to a Cobb-Douglas:

\[ v(x^2y^{3/5}) = x^{2/5}y^{3/5} \]

\[ x = \frac{a}{P_x} = \frac{2m}{5P_x} \]

Ben:

\[ \frac{\partial u}{\partial x} = 2(x+1)(y+2)^3 \quad \frac{\partial u}{\partial y} = 2(x+1)^2(y+2)^2 \]

\[ \frac{2(x+1)(y+2)^3}{(x+1)^2 3(y+2)^2} = \frac{P_x}{P_y} \] (simplify)

\[ \frac{2(y+2)}{3(x+1)^2} = \frac{P_x}{P_y} \] (solve for y)

\[ 2y = \frac{P_x}{P_y} \frac{3(x+1)}{2} - 4 \]

\[ y = \frac{P_x}{2P_y} \frac{3(x+1)}{2} - 4 \] (insert this into the budget restriction)

\[ m = P_x x + P_y \left( \frac{P_x}{P_y} \frac{3(x+1)}{2} - 4 \right) \] (simplify)

\[ m = P_x x + P_x \frac{3(x+1)}{2} - P_y 2 = P_x x + \frac{3}{2} P_x x + \frac{3}{2} P_x - 2 P_y \] (solve for x)

\[ P_x x + \frac{3}{2} P_x x = m - \frac{3}{2} P_x + 2P_y \]

\[ \frac{5}{2} P_x x = m - \frac{3}{2} P_x + 2P_y \] (divide both sides with \( \frac{5}{2} P_x \))

\[ x = \frac{2m}{5P_x} - \frac{3}{2} \frac{2P_x}{5P_x} + \frac{2}{5} \frac{P_y}{P_x} \]

\[ x = \frac{2m}{5P_x} - \frac{4 P_y}{5 P_x} \] (insert this into the expression for y)

\[ \frac{P_x}{P_y} = \frac{2m}{5} \frac{3}{5} - \frac{4 P_y}{5} + 1 \]

\[ y = \frac{P_y}{2} - 2 \]
\[ y = \frac{P_x 3}{P_y 2} \frac{2 m}{5 P_x} - \frac{P_x 3}{P_y 2} \frac{3}{5 P_x} + \frac{P_x 3}{P_y 2} \frac{4 P_y}{5} + \frac{P_x 3}{P_y 2} \frac{1}{2} - 2 \]  
(simplify)

\[ m = \frac{6}{P_y} \frac{P_x 9}{10} \frac{12}{10} + \frac{P_x 3}{P_y} \frac{8}{10} \]

\[ y = \frac{m 3}{P_y} + \frac{6}{10} \frac{P_x}{P_y} - \frac{8}{10} \]

\[ y = \frac{m 3}{P_y} + \frac{3 P_x}{5} - \frac{4}{5} \frac{P_y}{5} \]

**Problem 4. Demand**

C denotes the number of cups of coffee and B the number of buns:

a) \(15C+ 10B = 150\). The budget line intersects the “coffee-axis” at \(C = 10\) and the “bun-axis” at \(B = 15\).

b) 6 of each. Her indifference curves are L-shaped with the corners on the 45-degree line.

c) Perfect complements.

**Problem 1 Slutsky equation**

a. Value of endowment: \(500 + 50*50 = 3000\)

b. Connect the points \((0,0), (50, 0), (50, 500)\) and \((0, 3000)\)

c. d. and e. \(L = \text{Leisure} \quad Y = \text{non-labour income}\)

H= Labour supplied

Budget constraints: \(C = Y + wH\) and \(L = T - H\)

\[ C = Y + w(T - L) \quad \Leftrightarrow \quad C + wL = Y + wT \]

Maximise \(U(C,L) = CL\)

s.t. \(C + wL = Y + wT\)

MRS: \(\frac{MU_C}{MU_L} = L \quad \text{and} \quad \frac{MU_C}{MU_L} = C \implies MRS = \frac{-C}{L}\)

Set MRS equal to the slope of the budget line:

\[ \frac{C}{L} = \frac{w}{1} \quad \text{C} = wL \]

Insert this into the budget restriction:

\[ C + C = Y + wT \]

\[ C = \frac{Y}{2} + \frac{wT}{2} \quad \text{which solves part d.} \]
or \( wL + wL = Y + wT \) which gives us \( L = \frac{Y}{2w} + \frac{T}{2} \) and solves e.

With \( T = 50, w = 50 \) and \( Y = 500 \) we get the answer to b. as \( L = 5 + 25 = 30 \) and labour supply \( = 50 - 30 = 20 \) while \( C = 500 + 20 \times 50 = 1500 \)

def. Labour supplied if \( Y = 0 \):

\[
L = \frac{50}{2} = 25 \text{ and labour supply } H = 50 - 25 = 25
\]

**Problem 2 Slutsky equation**

a. Lagrange function: \( L(x, y, \lambda) = x^{0.3}y^{0.5} - \lambda (5x + 8y - 160) \)

First order conditions for maximum:

\[
0.3 x^{-0.7}y^{0.5} - 5 \lambda = 0 \\
0.5 x^{0.3}y^{-0.5} - 8 \lambda = 0 \\
5x + 8y - 160 = 0
\]

Piecewise division of the first two gives \( x = 0.96y \), inserting into the constraint gives \( x = 12, y = 12.5 \)

(You could also have taken the demand functions with a Cobb-Douglas utility function as known.

\[
x = \frac{0.3 \times 160}{0.8 + 5} \quad \text{and} \quad y = \frac{0.5 \times 160}{0.8 + 8}
\]

\( \text{MRS} = \frac{MUx}{MUy} = 0.3 x^{-0.7}y^{0.5} / 0.5 x^{0.3}y^{-0.5} = 0.6y/x = 0.6 \times 12.5 / 12 = 0.625 \)

\( p_x / p_y = 5/8 = 0.625 \)

b) The MRS shows the maximum amount of y that she could trade for one unit of x, without losing utility. If it is lower than the relative price of x, she would be better off consuming less of x and more of y. If it is higher than the relative price of x, she would be better off consuming more of x and less of y.

For example, if \( MUx = MUy = 1 \) and the price of x is twice that of y, she could give up one unit of y and get two units of x, gain two units of utility and give up one unit. If the price of y is twice the price of x, she could give up one unit of y (and one unit of utility) and get two units of x (and two units of utility).

c) i. The old budget constraint intersects the axes at (0, 20) and (32, 0), the new at (0, 0) and (40, 0)

ii. \( x = 15, y = 12.5 \)

iii. \( m' = 148 \)

iv. substitution effect: \( x(4, 148) - x(5, 160) \approx 13.9 - 12 = 1.9 \)

income effect: \( x(4, 160) - x(4, 148) \approx 15 - 13.9 = 1.1 \)
Problem 1. Consumer’s surplus

a) \( P = 30 - 2B \)

b) Willingness to pay: \( 10 \times 10 + \frac{(30-10) \times 10}{2} = 200 \)

Consumer’s surplus: \( \frac{30-20}{2} \times 10 = 100 \)

Problem 2. Consumer’s surplus

\( U(x, y) = x^{0.1} y^{0.9} \)

\( m_0 = 100, \ p_0 = q_0 = 1 \) where \( p \) is the price of \( x \), \( q \) the price \( y \)

Calculate CV and EV when \( p \) increases to 2

i) Initially \( U \) is a Cobb-Douglas function so we know that

\( x = \frac{am}{p} = \frac{0.1 \times 100}{1} = 10 \)

\( y = \frac{bm}{q} = \frac{0.9 \times 100}{1} = 90 \)

\( U = 10^{0.1} \cdot 90^{0.9} \)

ii) If \( p = 2 \) then \( x = 0.1 \times 100 \times 1/2 = 5 \)

\( y \) is independent of \( p \), and therefore unchanged.

\( U = 5^{0.1} \cdot 90^{0.9} \)

iii) CV: Let \( m' = 100 + CV \) and assume that \( p = 2 \)

\( x = 0.05m' \)

\( y = 0.9m' \)

The definition of CV \( \Rightarrow \)

\( U \left( \frac{0.1m'}{2}, \frac{0.9m'}{1} \right) = U(10,90) = 10^{0.1}90^{0.9} \)

\( \frac{0.1}{2^{0.1}}(m')^{0.1} \cdot 0.9^{0.9}(m')^{0.9} = 10^{0.1}90^{0.9} \)

\( (m')^{0.1+0.9} = \frac{10^{0.1} \cdot 2^{0.1} \cdot 100^{0.9} \cdot 0.9^{0.9}}{0.1^{0.1} \cdot 0.9^{0.9}} \)

\( m' = \left( \frac{10}{0.1} \right)^{0.1} \cdot 2^{0.1} \cdot 100^{0.9} = 2^{0.1} \cdot 100^{0.1+0.9} = 2^{0.1} \cdot 100 \approx 107,177 \)

\( CV = 7,18 \)

iv) EV: Find the income \( m' = 100 - EV \) such that the maximal utility when \( p=2 \) and \( m=100 \) is the same as when \( p=1 \) and \( m = m' \)

If \( p = 2 \) and \( m = 100, \ U = 5^{0.1}90^{0.9} \) according to ii)
If \( p = 1, m = m' \) then \( x = 0,1m' \) och \( y = 0,9m' \)

\[
U = (0,1)^{0,1} \cdot (m')^{0,1} \cdot (0,9)^{0,9} \cdot (m')^{0,9} = m'(0,1)^{0,1} \cdot (0,9)^{0,9} = 0,1 \cdot (9)^{0,9} \cdot m'
\]

should be equal to \( 5^{0,1} \cdot 9^{0,9} \)

After simplification, this implies that \( m' = 100 \cdot 2^{-0,1} \approx 93,30 \)

\( EV \approx 6,7 \)

**Problem 3. Consumer’s surplus**

The consumer surplus is a (monetary) measure of the utility a consumer derives from her/his consumption of the good and the price paid for it and it corresponds to the area under the demand curve, but above the price (assuming that the income effect is so small that it can be disregarded).

\( p^* \) and \( q^* \) are the price and the quantity demanded at this price.

If you see the demand curve as that of an individual, the reason for CS is that the reservation price is decreasing in \( q \) and the person would have been willing to pay more if the seller could have charged the highest possible price for each unit. If you see it as the market, aggregate demand different individuals have different reservation prices and - since perfect price discrimination is usually not possible – the market price for all buyers will be the reservation price of the marginal buyer. (Either answer is OK.)

**Problem 4. Consumer’s surplus**

a) \( 5 \cdot 50 + (100-50) \cdot 5/2 = 375 \). (Hint: It is easier to see this if you plot the demand curve in a figure.)

b) \( 375 - (5 \cdot 50) = 125 \)

**Problem 5. Consumer’s surplus**

a) To ask for the compensating variation: What compensation (for example, in the form of a tax decrease) would make you agree to have the new housing construction?

b) To ask for the equivalent variation: How much would you be willing to give up (say, in the form of a tax increase) if this could stop the construction plans?
Problem 1. Market demand

Demand function \( D(p) = q = 10 - 2p \)

a) What is the price elasticity of demand when \( p = 3 \)

\[
\frac{dq}{dp} = -2 \\
q(3) = 10 - 2 \times 3 = 4
\]

\[
\epsilon = -2 \times \frac{4}{4} = -3/2
\]

b) At what price and quantity is \( \epsilon = -1 \)?

\[
\epsilon = -1 \implies \frac{dq}{dp} \cdot \frac{p}{10 - 2p} = -1
\]

\[
-2 \cdot \frac{p}{10 - 2p} = -1
\]

\[
2p = 10 - 2p
\]

\[
p = \frac{10 - 2p}{2} = 5 - p
\]

\[
p = 2.5
\]

c) If the demand function is \( D(p) = a - bp \), what is the elasticity of demand?

\[
\epsilon = \frac{dq}{dp} \cdot \frac{p}{q} = -b \frac{p}{a - bp}
\]

Problem 2. Market demand

\( q(p) = (p+1)^2 \)

a) The price elasticity of demand

\[
\epsilon = \frac{dq}{dp} \cdot \frac{p}{q} = -2(p+1)^{-3} \cdot \frac{p}{(p+1)^{-2}} = -2p(p+1)^{-3} (p+1)^2 = -\frac{2p}{p+1}
\]

b) If \( p \neq -1 \), \( \frac{-2p}{p+1} = -1 \iff -2p = -p - 1 \iff p = 1 \)

c) Total revenue \( R = pq = p(p+1)^2 \)

d) Answers to a) - c) when \( q(p) = (p+a)^b \), \( a > 0 \) and \( b < 1 \)

\[
\epsilon = \frac{bp}{p+a} \implies \epsilon = -1 \text{ when } p = -a/(b+1) \quad R(p) = p(a+p)^b
\]

Problem 3. Market demand

a) \( \frac{dD(p)}{dp} = -6 \) and \( \frac{p}{q} = \frac{p}{(30-6p)} \), so \( \epsilon = -6p/(30-6p) \)
b) $\epsilon=\frac{-p}{(60-p)}$

c) $\epsilon=\frac{-bp}{(a-bp)}$

d) $\epsilon=-2$

e) $\epsilon=-b$

f) $\epsilon=\frac{-2p}{(p+3)}$

**Problem 1. Equilibrium**

a. $D(p) = S(p)$

\[
200 - p = 150 + p \\
p = 25 \\
D(p) = 200 - 25 = 175
\]

b. When is $S = 160$:

\[
160 = 150 + p \\
p = 10
\]

c. Willingness to pay at $q = 160$:

\[
160 = 200 - p \\
p = 40
\]

Maximum price - minimum price = 40 - 10 = 30 = price of the coupons.

d. Dead-weight loss of restricting the supply:

\[
DW = \frac{30 \cdot (175-160)}{2} = 225.
\]

**Problem 2. Equilibrium**

$q^D = 100 - 5p \quad q^S = 5p$

$p^S = \frac{q^S}{5}$

No tax:

\[
100 - 5p = 5p \\
p = 10 \\
q = 50
\]

Tax:

\[
p^S = \frac{q^S}{5} + 2 \implies q = 5p - 10 \\
100 - 5p = 5p - 10 \\
p = 11
\]
\[ q = 45 \]

The revenue per unit is (according to the old supply curve):
\[ p = \frac{45}{5} = 9 \]

N.B. the price increase is not equal to the tax.

**Problem 3. Equilibrium**

a) With \( p \) on the vertical axis and \( q \) on the horizontal: The demand curve intersects the \( p \)-axis above the \( q \)-axis and slopes downwards to the right, the supply curve slopes upwards to the right. It ends at a point with \( p = 0 \) and \( q > 0 \) because \( p < 0 \) is not economically feasible. If the line had continued, it would have intersected the \( p \)-axis at a value \( p < 0 \).

b) See Varian, figure 16.3. The demand curve shifts downwards by the amount \( t \), the supply curve is unchanged. \( q \) is reduced, the consumer pays a higher price and the producer gets a lower price than before. There is a loss of welfare, due to the deadweight loss (reduction in \( q \)). The amount of the tax is a welfare loss to consumer and/or producer but not a social welfare loss since it is a transfer that can be used for other welfare-increasing purposes. (Which may even compensate some deadweight loss if it is used to rectify a market imperfection somewhere else in the economy or to make distribution of wealth agree better with the social welfare function, which we will cover later on in this course.)

c. See Varian figure 16.3. The results are exactly the same as in b. How much of the incidence of the tax is born by consumers and producers, respectively, depends on the slopes of the demand and supply curves, not on who formally pays the tax.

**Problem 1. Intertemporal choice**

a. \( C_2 = m_2 + (1+r)(m_1 - C_1) \)

Maximum consumption in period 1:
\[
200.000 + \frac{200.000}{1 + r} = 200.000 + \frac{200.000}{1+2} = 266.667
\]

If she borrows 66.667 she will have to pay \((1+r)\) 66.667 in period 2.

Maximum consumption in period 2:
\[
200.000 + 200.000 = 400.000
\]

b. New endowment: \( m_1 = 300.000 \) and \( m_2 = 150.000 \).

Maximum consumption in period 1:
\[
300.000 + \frac{150.000}{3} = 350.000
\]
Maximum consumption in period 2:

\[300.000 + 150.000 = 450.000\]

The consumer will be better off. The consumption that she did choose before the change in the endowment is still affordable.

c. \(m_1 = 150.000\) and \(m_2 = 300.000\)

Maximum consumption in period 1:

\[150.000 + \frac{300.000}{3} = 250.000\]

Maximum consumption in period 2:

\[150.000 + 300.000 = 450.000\]

b is better or at least as good as c.

If he is a saver then c is better than a. Better otherwise we cannot say if a or c is the better alternative.

**Problem 2. Intertemporal choice**

a. \(PV = 300000\)

b. Future value = 330000

c. \(U = C_1 C_2\)

\[\text{MRS} = \frac{C_2}{C_1}\]

d. The budget constraint:

\[C_2 = m_2 + (1+r)(m_1 - C_1)\]
\[C_2 = m_2 + (1+r)m_1 - (1+r)C_1\]

Slope: \[\frac{C_2}{C_1} = -(1+ r) = -(1 + 0.1)\]

Set this equal to the MRS:

\[-\frac{C_2}{C_1} = -(1+0,1) \ (\text{solve for } C_2)\]

\[C_2 = 1,1 \times C_1\]

(insert this into the budget constraint)

\[1,1C_1 = 110.000 + (1+0,1)200.000 - (1+0,1)C_1\]  \hspace{1cm} (solve for \(C_1\))

\[2,2C_1 = 330.000\]

\[C_1 = 150.000\]  \hspace{1cm} (insert this into the expression for \(C_2\))

\[C_2 = 1,1 \times 150.000 = 165.000\]

She will save (200.000 - 150.000) 50.000 in the first period.
e. If \( r = 20\% \) then \( C_2 = 1.2C_1 \) (insert this into the budget constraint)

\[
1.2C_1 = 110.000 + (1+0.2)200.000 - 1.2C_1 \quad \text{(solve for } C_1)\]

\[
C_1 = 145.833 \\
C_2 = 1.2 \times C_1 = 175.000
\]

Utility in d: \( U = 150.000 \times 165.000 = 24.750.000 \)

Utility in e: \( U = 145.833 \times 175.000 = 25.520.775 \)

She is better off if the interest rate is 20%.

**Problem 1. Uncertainty**

a. \( U_{\text{No game}} = \sqrt{100.000} = 316.2 \)

\( U_{\text{Game}} = 0.5 \sqrt{90.000} + 0.5 \sqrt{110.000} = 315.8 \)

Jonas doesn’t take the bet.

Declining marginal utility for money ==> risk aversion.

b. \( C_1 = 0 \quad C_2 = 200.000 \)

\( U_{\text{Game}} = 0.5 \sqrt{200.000} = 223.6 \)

Jonas doesn’t take the bet, since \( U_{\text{No game}} > U_{\text{Game}} \)

c. \( C_1 = 0 \quad C_2 = 600.000 \)

\( U_{\text{Game}} = 0.5 \sqrt{600.000} = 387.3 \)

Jonas take the bet, since \( U_{\text{Game}} > U_{\text{No game}} \)

d. \( C_1 = 0 \quad C_2 = X \)

\( U_{\text{Game}} = 0.5 \sqrt{0} + 0.5 \sqrt{X} \)

\( U_{\text{No game}} = \sqrt{100.000} \)

Set this two equal to find the value of \( X \) where Jonas is indifferent between taking the bet or not:

\[
0.5 \sqrt{0} + 0.5 \sqrt{X} = \sqrt{100.000}
\]
That is: "If you win I will give you 300.000."

**Problem 2. Uncertainty**

a. \( U_{\text{No game}} = 1.000^2 = 1.000.000 \)

\[ C_1 = 800 \quad C_2 = 1.200 \quad \alpha_1 = 0.75 \quad \alpha_2 = 0.25 \]

\[ U_{\text{Game}} = 0.75 \times 800^2 + 0.25 \times 1.200^2 = 840.000 \quad < 1.000.000 \]

b. \( U_{\text{Game}} = 0.75 \times 0^2 + 0.25 \times 2.000^2 = 1.000.000 \)

He is indifferent between taking the bet or not.

c. \( U_{\text{Game}} = U_{\text{No game}} \)

\[ 0.75 \times C_1^2 + 0.25 \times C_2^2 = 1.000.000 \]

\[ \begin{array}{cc} C_2 & C_1 \\ 0 & 1154,7 \\ 500 & 1118 \\ 1.000 & 1.000 \\ 1.500 & 763,8 \\ 2.000 & 0 \end{array} \]

d. \( U_{\text{Game}} = U_{\text{No game}} \)

\[ 0.5 \times C_1^2 + 0.5 \times C_2^2 = 1.000.000 \]

\[ C_1 = \sqrt{2.000.000 - C_2^2} \]

\[ \begin{array}{cc} C_2 & C_1 \\ 0 & 1.414,2 \\ 500 & 1.323 \\ 1.000 & 1.000 \\ 1.414,2 & 0 \end{array} \]

**Problem 3. Uncertainty**

Not if risk averse. Yes, if risk lover.